Pair creation and decay of a massive particle near and far away from a cosmic string

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(Received 21 May 1991)

We study the total transition probabilities of the tree-level processes of the pair creation and decay of a massive particle for real Klein-Gordon fields in the spacetime of an infinite straight static cosmic string. Basing the discussion on cylindrical modes characterized by an approximate radius of closest approach $r_{\min}$, it is possible to approximately localize the non-Minkowskian processes to cylindrical effective interaction regions around the cosmic string. A physical understanding of the space dependence of the transition probabilities is obtained on the basis of analytic expressions for different energy domains referring to regions close to and far away from the string. For pair creation the Compton wavelength λ_C of the created particles proves to be a crucial length scale. For $r_{\min} \ll \lambda_C$ the creation probability is insensitive to a variation of $r_{\min}$. For large $r_{\min}$ it falls off at least exponentially with $r_{\min}$. This agrees with an alternative "integrated" approach to localization: the cross section around the cosmic string is proportional to the Compton wavelength λ_C. The decay of the massive particle on the other hand contains processes allowed in Minkowski spacetime and leads to another type of local behavior.

PACS number(s): 98.80.Cq, 04.60.+n, 11.20.Dj

I. INTRODUCTION

Cosmic strings may have been created during the phase transitions in the early Universe [1]. The spacetime of a static gauge cosmic string is locally flat everywhere except at the string, but possesses a conical global structure [2]. The simplest possible string is an infinite straight one which is infinitely thin. Most of the characteristic classical and quantum effects, which are due to the non-Minkowskian topology of cosmic strings, already show up in this case. For the classical ones, which have been extensively discussed in the literature, we refer to the review articles of Refs. [3] and [4]. From the quantum-field-theoretical point of view, the nontrivial topology leads to a polarization of the quantum vacuum. The corresponding vacuum expectation value of the stress-energy-momentum tensor is very similar to the one in the well-known Casimir effect [5–10]. Also, particle detectors respond differently when they move in the vicinity of a cosmic string [11]. Furthermore one may note that, because of the presence of the string, there is a breakdown of the translational invariance in the plane perpendicular to the string leading to nonconservation of the related linear momentum [12–14]. On the other hand, quantum-field-theoretical processes are by their very nature nonlocal processes. They are sensitive to the presence of the string and therefore this breakdown of the global translational invariance has important consequences on the quantum process. One consequence is that processes of mutual interaction which are not allowed in empty Minkowski spacetime may happen in a string spacetime [12–14]. In addition, transition probabilities of processes which are allowed in flat spacetime are expected to be modified, because transitions into new momentum channels are possible in the presence of strings [14].

Although quantum-field-theoretical processes are nonlocal, one expects from the physical point of view that the actual decay or scattering of a particle is effectively a somewhat localized process, so that transition probabilities should approach the respective Minkowskian values in the asymptotic regions far away from the string and should, on the other hand, reflect the presence of the topological influence of the string if the process happens near the string. For the discussion of the physical influence of strings in the cosmological context it is important to obtain an idea of the characteristic range of influence of a string and the spatial dependence in detail. This physical problem adds to the usual problem of the evaluation of the functional structure of transition probabilities, decay rates and so on in the presence of the cosmic string.

In this paper both types of problems are studied for the case of pair creation and the decay of a massive particle. The interaction between the massless and massive real scalar fields is thereby introduced for the pair creation by a toy Lagrangian with structure analogous to the Lagrangian of quantum electrodynamics (QED). The corresponding QED calculations are structurally similar and are expected to reflect the same physical conclusions. We must stress here again that in this paper we are interested in the effects of the nontrivial topology of the cosmic strings on quantum processes outside the string's core. Therefore we shall not consider any coupling of the external quantum fields to the gauge or Higgs fields that make up the string's core. For such kinds of interaction we refer to Ref. [15] and the references therein, where the scattering of fermions from cosmic strings is studied, without however taking into account the conical structure of the spacetime.

The treatment below is based on the results of a previous paper [14], where the transition probabilities of tree-level processes in cosmic-string spacetimes are discussed in general for Minkowskian and non-Minkowskian processes. In Ref. [14] using the property of the localized absence of cylindrical modes we have shown that it is possible to localize processes of mutual interaction to cylindrical effective interaction regions around the cosmic string. In order to discuss the consequences of this on the space dependence of transition probabilities, particular processes have to be specified and the respective transition probabilities have to be worked out in detail. This is done below for two important process which show typical differences from the physical point of view.

In Sec. II we fix the notation, characterize the procedure, and summarize the main results of Ref. [14]. The mathematical treatment and the physical results depend crucially on whether the measures of the momenta in the plane perpendicular to the string satisfy the triangle inequalities or not. The respective characterization for three types of tree-level processes is given in Sec. III. In Sec. IV we study the total transition probability for pair creation and give analytic expressions in the low- and high-energy limit in regions far away from and close to the string. Alternative approaches support the physical discussion of the localization. We complete the discussion in Sec. IV by studying the process of the decay of a massive particle with regard to the same questions. In this case special attention is given to the decay channels which satisfy the triangle inequalities.

II. BASIC FORMULAS

The metric for an infinite straight, static, gauge cosmic string located at the z axis is [2]

$$ds^2=dt^2-dz^2-dr^2-b^2r^2d\varphi^2 , \tag{2.1}$$

where $0\leq\varphi<2\pi, 0<r<\infty$, and $z,t\in(-\infty,+\infty)$ while the constant $b\in(0,1]$. For realistic cosmic strings of grand-unified-theory (GUT) scale, called GUT cosmic strings, the relevant values of b are $(1-b)\approx 10^{-5}\ll 1$. In the limit of $b=1$ we have the Minkowski spacetime in polar coordinates. We use throughout this paper units where $\hbar=c=1$.

Let ϕ and ψ be Klein-Gordon scalar fields in the spacetime represented by (2.1) which are mutually interacting via the interaction Lagrangian density

$$\mathcal{L}_I=-\lambda\phi\psi^2 , \tag{2.2}$$

where λ is a coupling constant with dimension of mass. The interaction (2.2) belongs to the type of interactions that we have studied in Ref. [14], hereafter called paper I. In the present work we will apply the results of paper I to the processes of pair creation and decay of a massive particle. To fix the notation and simplify the reference we summarize below the main results which are used in the following sections. For details the reader is referred to paper I.

A. Particle states

In our discussion we will mainly use particle states $|1_{\kappa l\xi}\rangle$ with fixed z momentum κ, z angular momentum lb^{-1}, and measure of the $x-y$ momentum ξ. The construction of such states is based on the complete set of cylindrical mode solutions $u_{\kappa l\xi}$ of the field equations

$$u_{\kappa l\xi}:=u_j=(2b)^{-1/2}e^{i\kappa z}e^{il\varphi}J_{|l|/b}(\xi r)e^{-iE_jt} . \tag{2.3}$$

Here $J_\nu(z)$ are Bessel functions, while $E_j=(\xi^2+\kappa^2+m^2)^{1/2}$ is the energy, and m is the mass of a particle. j denotes the collective quantum label $j=\{\kappa,l,\xi\}$ with $l\in\mathbb{Z},\kappa\in(-\infty,+\infty)$, and $\xi\in(0,+\infty)$.

The cylindrical modes u_j in (2.3) are normalized to a radial ingoing and an equal radial outgoing flow of one particle per unit time, per unit z length of a cylindrical surface at radial infinity. For sufficiently large radial distances $r\gg|l|/(b\xi)$, the particle density is in the average equal to $E_j/(\pi b r\xi)$ independent of l. For physical interpretations, compare paper I for a configuration of classical particles which represent the classical counterpart of the quantum modes u_j.

An important property of the u_j modes is their *localized absence:* the probability of finding the particle in radial distances smaller than the classical radius of closest approach $r_{\min}(\kappa,l,\xi)=|l|/(b\xi)$, is very small and decreases rapidly as $|l|$ increases.

B. Total transition probabilities

The processes with which we will deal in this paper are transitions from a one-particle state to a two-particle one. Let $j_1=\{\kappa_1,l_1,\xi_1\}, j_2=\{\kappa_2,l_2,\xi_2\}$ be the quantum numbers in the final state and $j=\{\kappa,l,\xi\}$ in the initial state while m_1,m_2,m and E_1,E_2,E are their corresponding masses and energies. If the final particles are not identical then the corresponding total probability, within an infinite time interval T and infinite z length L, is

$$w(\kappa,l,\xi):=w(j)=\frac{\lambda^2\pi^2}{2b}\left[\frac{TL}{4\pi^2}\right]\int d\xi_1 d\xi_2\,\xi_1\xi_2\Sigma_1\Sigma_2 . \tag{2.4}$$

Here

$$\Sigma_2\equiv\int d\kappa_1 d\kappa_2\frac{\delta(\kappa-\kappa_1-\kappa_2)\delta(E-E_1-E_2)}{E_1E_2} = \frac{4H(\hat{\xi}-\hat{\xi}_1-\hat{\xi}_2)}{\sqrt{s}} , \tag{2.5}$$

where H is the step function and

$$s=[\hat{\xi}^2-(\hat{\xi}_1+\hat{\xi}_2)^2][\hat{\xi}^2-(\hat{\xi}_1-\hat{\xi}_2)^2] , \tag{2.6}$$

with

$$\hat{\xi}^2=\xi^2+m^2,\quad \hat{\xi}_1^2=\xi_1^2+m_1^2,\quad \hat{\xi}_2^2=\xi_2^2+m_2^2 . \tag{2.7}$$

In Minkowski spacetime the $x-y$ momentum vectors form a triangle. Accordingly their measures satisfy the triangle inequalities

$$|\zeta_1-\zeta_2|<\zeta<\zeta_1+\zeta_2 \; . \tag{2.8}$$

In cosmic-string spacetimes processes without $x-y$ momentum conservation, called non-Minkowskian processes, may happen. Although now their measures may in particular cases still satisfy the triangle inequality (2.8), in general this will not be the case. Mathematically the processes obeying and, respectively, not obeying Eq. (2.8) must be treated separately. Note that the first case may comprise Minkowskian and non-Minkowskian processes. The physical results will reflect these distinctions.

If (2.8) is not satisfied, we find, for Σ_1 of Eq. (2.4),

$$\Sigma_1=\begin{cases}\sin(\pi/b)(e^{-2|l|w_1/b}+e^{-2|l|w_2/b})F & \text{if } \zeta>\zeta_1+\zeta_2 \; ,\\ [\sin(\pi/b)-G]e^{-2|l|w_2/b}F & \text{if } \zeta_1>\zeta+\zeta_2 \; ,\\ [\sin(\pi/b)-G]e^{-2|l|w_1/b}F & \text{if } \zeta_2>\zeta+\zeta_1 \; ,\end{cases} \tag{2.9}$$

where

$$F=\frac{\coth(w/b)\sin(\pi/b)}{16\pi^2\mathcal{A}^2[\sinh^2(w/b)+\sin^2(\pi/b)]} \; ,$$
$$G=\frac{1}{2\cosh(w/b)}\left[\frac{\sin[(2|l|+1)\pi/b]}{e^{(2|l|-1)w/b}}-\frac{\sin[(2|l|-1)\pi/b]}{e^{(2|l|+1)w/b}}\right] \tag{2.10}$$

and

$$\cosh w=\frac{|\zeta_1^2+\zeta_2^2-\zeta^2|}{2\zeta_1\zeta_2}, \quad \zeta_1\zeta_2\sinh w=\zeta\zeta_2\sinh w_1=\zeta\zeta_1\sinh w_2=:2\mathcal{A} \; . \tag{2.11}$$

On the other hand Σ_1 turns out to be infinite in the case where the triangle inequalities (2.8) are satisfied. This reflects the presence of Minkowskian processes. To normalize the result, we confine the system within a cylinder of radius $R\to\infty$ with the axis of symmetry coinciding with the cosmic string and find that

$$\Sigma_1=\frac{b\gamma R}{4\pi^2\zeta\mathcal{A}} \quad \text{if } |\zeta_1-\zeta_2|<\zeta<\zeta_1+\zeta_2 \; , \tag{2.12}$$

where γ is some dimensionless proportionality factor which is left unspecified by our normalization procedure. In Eq. (2.12), $\mathcal{A}$ is defined as

$$2\mathcal{A}:=\zeta_1\zeta_2\sin\theta, \quad \cos\theta=\frac{\zeta_1^2+\zeta_2^2-\zeta^2}{2\zeta_1\zeta_2} \; . \tag{2.13}$$

If the final particles are identical then an additional multiplicative factor $\frac{1}{2}$ is to be understood in Eq. (2.4).

C. Localization

The quantity $\delta w(r_l;\zeta,\kappa)$, defined as

$$\delta w(r_l;\zeta,\kappa):=b\zeta[w(\kappa,l,\zeta)-w(\kappa,l+1,\zeta)]_{l=r_lb\zeta}\delta r_l \; , \tag{2.14}$$

can be approximately interpreted as the average probability that the transition of the initial particle with quantum number $\{\kappa,l,\zeta\}$ to a two-particle state happens within the region $r\in(r_l,r_l+\delta r_l)$ where $r_l\sim|l|/(b\zeta)$ and $\delta r_l\sim 1/(b\zeta)$. This interpretation is based on the observation that the modes (2.3) representing the initial states $|1_{\kappa l\zeta}\rangle$ and $|1_{\kappa,l+1,\zeta}\rangle$ are essentially different only within the region $(r_l,r_l+\delta r_l)$. Compare paper I for further details.

The formal equality

$$w(\kappa,l',\zeta)=\sum_{l=l'}^{\infty}\delta w(r_l;\zeta,\kappa) \; , \tag{2.15}$$

gives, via the dependence of r_l, a rough quantitative estimate for the r dependence of the local distribution of the transition probabilities $w(\kappa,l',\zeta)$.

D. Cross sections

The usual definition of cross sections relies on the use of particle states which behave asymptotically as plane waves. The total transition probability $w(\mathbf{p}):=w(1_\mathbf{p}\to 2)$, from such a plane-wave initial state $|1_\mathbf{p}^+\rangle$, which is labeled by the asymptotic momentum $\mathbf{p}$ and is normalized (in the in region) to a particle density $(8\pi^3b)^{-1}$, is related to the total transition probabilities $w(\kappa,l,\zeta)$ by the simple relation

$$w(\mathbf{p})=\frac{1}{8\pi^3E_j}\sum_{l=-\infty}^{+\infty}w(\kappa,l,\zeta) \; . \tag{2.16}$$

Here $p_z=\kappa$, $\zeta^2=p_x^2+p_y^2$, and $E_j=(m^2+\zeta^2+\kappa^2)^{1/2}$.

For processes that are forbidden in Minkowski spacetime, the "target" or the "scatterer" is clearly the cosmic string. Furthermore, since the incoming flux which corresponds to the state $|1_\mathbf{p}^+\rangle$ is $(8\pi^3b)^{-1}(\zeta/E_j)$, the cross section $\sigma(\mathbf{p})$ per unit length around the string is defined as

$$\sigma(\mathbf{p}):=\frac{w(\mathbf{p})}{TL(8\pi^3b)^{-1}(\zeta/E_j)}=\sum_l\frac{b}{\zeta}\frac{w(\kappa,l,\zeta)}{TL}$$
$$=:\sum_l\sigma_l(\kappa,\zeta) \; . \tag{2.17}$$

Equation (2.16) is also the defining relation for the partial cross sections $\sigma_l(\kappa,\zeta)$.

The above definition of cross section is not applicable in the case where the realization of the process is not attributed to the cosmic string alone. The reason is that now the number of "scatterers" is not one (i.e., only the cosmic string), as above. The definition of a cross section is not straightforward in this case since the specification of the density of scatterers is not obvious.

III. TYPES OF TREE-LEVEL PROCESSES

In this and a subsequent paper we will study the following tree-level processes of the interaction (2.2), which are transitions from a one-particle state to a two-particle one:

$$\text{Pair creation (PC):}\quad |1_j^\phi\rangle \to |1_{j_i}^\psi 1_{j_2}^\psi\rangle,\quad \{m^\phi, m^\psi\}=\{0,M\}\ ,$$

$$\text{Decay (D):}\quad |1_j^\phi\rangle \to |1_{j_1}^\psi 1_{j_2}^\psi\rangle,\quad \{m^\phi, m^\psi\}=\{M,0\}\ ,$$

$$\text{Bremsstrahlung (BS):}\quad |1_j^\psi\rangle \to |1_{j_1}^\psi 1_{j_2}^\phi\rangle,\quad \{m^\phi, m^\psi\}=\{0,M\}\ ,$$

$$\text{Approximate BS}(\approx\text{BS}):\quad |1_j^\psi\rangle \to |1_{j_1}^\psi 1_{j_2}^\phi\rangle\quad \{m^\phi, m^\psi\rangle=\{\mu,M\},\quad \mu\to 0^+\ .$$

Here the indices ϕ,ψ are used to denote quantities referring to the fields ϕ and ψ, respectively.

In the case of the PC and BS processes, the interaction Lagrangian density is given by Eq. (2.2) with $m^\phi=0$, $m^\psi=M$. It is similar (but of course, not identical) to the quantum electrodynamical one $\mathcal{L}_{\rm QED}=-e\bar\psi\gamma^\mu A_\mu\psi$. Parts of the respective calculations can easily be transcribed. In $\mathcal{L}_{\rm QED}$, the coupling constant e is the charge of the electron, ψ is the fermionic field of mass M, A_μ is the massless electromagnetic field, and γ^μ are the Dirac matrices. The "mapping" of our scalar interaction Lagrangian to the QED one is given by $\phi\to A_\mu$, $\psi\to\psi$, and $\lambda/2M\to e$. However differences should be taken into account. For example, the spin-1 photon field A_μ has two polarization states in contrast with its scalar counterpart ϕ.

The study of the processes above covers essentially all the physical characteristics which are to be expected for transitions from a one-particle state with (κ,l,ζ) to a two-particle one with (κ_1,l_1,ζ_1) and (κ_2,l_2,ζ_2). This becomes evident if one considers the space of (ζ_1,ζ_2) pairs in which the momentum quantum numbers ζ_1 and ζ_2 can be combined according to the restrictions discussed below which are relevant for a given value of ζ and a particular physical process. Figure 1 shows this scaled ζ_1-ζ_2 plane.

In a cosmic-string spacetime all possible transitions are restricted by the energy and the z-momentum conservation [see Eq. (2.5)] which amount in our case to the condition

$$\hat\zeta\ge\hat\zeta_1+\hat\zeta_2\ . \tag{3.1}$$

In Fig. 1 we have plotted the curves $\hat\zeta=\hat\zeta_1+\hat\zeta_2$ for the PC, D, and BS processes which differ according to the mass values of the particles involved. Processes which are allowed according to the condition (3.1) are restricted to the regions between the axes and below the corresponding curves.

As stressed above, the mathematical treatment and the physical results depend crucially on whether or not the ζ quantum numbers satisfy the triangle inequalities (2.8). Processes that satisfy (2.8), as the Minkowskian processes necessarily do, are located in the hatched region in Fig. 1.

D processes can happen with conservation of the x-y momenta. Accordingly the allowed region consists partly of a region where the triangle inequalities hold. Note that the points there may refer also to processes with momentum measures which satisfy the triangle inequalities (2.8) but have nevertheless no conservation of the x-y momenta, because the respective vectors do not form a triangle. For D processes there are, in addition, regions allowed for which Eq. (2.8) does not hold. The PC, BS, and $\approx$ BS processes all take place with momentum nonconservation since for all of them we have $\zeta>\zeta_1+\zeta_2$ and according violation of Eq. (2.8). This can be seen from Fig. 1 or directly from eq. (3.1). For later reference we note that, although they are somewhat similar to the PC processes, the BS processes include processes which come arbitrarily close to the point ($\zeta_2=0$, $\zeta_1=\zeta$) which lies on the separation line to the triangle inequality satisfying

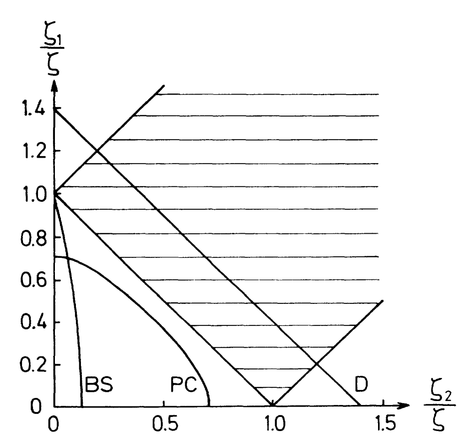

FIG. 1. The allowed region of ζ,ζ_1,ζ_2 parameters for the pair creation (PC), bremsstrahlung (BS), and the decay process (D) is the area between the axes and below the respective curves. The plotted curves refer to the masses (PC: $m^\psi=0.25\zeta$), (BS: $m^\psi=3\zeta$), (D: $m^\phi=1.4\zeta$). In the hatched region the ζ parameters satisfy the triangle inequalities (2.8).

cases. In the approximate bremsstrahlung process ($\approx$BS) this point is avoided in introducing a small mass μ for the "radiated" away ϕ particle. In Fig. 1 the respective curve would agree with the BS curve except near $\zeta_2 \approx 0$, where it reaches the ζ_1 axis at a point below $\zeta_1 = \zeta$. All these characteristic properties of the different processes will have consequences when working out and discussing the total transition probabilities.

In place of ζ_1, ζ_2 we introduce for technical reasons the u, v variables via

$$u = (\zeta_1^2 + \zeta_2^2)/\zeta^2, \quad v = (\zeta_1^2 - \zeta_2^2)/\zeta^2 . \tag{3.2}$$

In terms of these variables the characteristic s function of Eq. (2.6) reads

$$\begin{aligned} s = {} & (\zeta^2 + m^2 - m_1^2 + m_2^2)^2 \\ & -4m_1^2 m_2^2 + 2(m_1^2 - m_2^2)\zeta^2 v \\ & + \zeta^4 v^2 - 2\zeta^2(\zeta^2 + m^2)u , \end{aligned} \tag{3.3}$$

where m is the mass of the initial-particle and m_1, m_2 the mass of the final-particle states. For the PC, D, BS, and $\approx$BS processes the corresponding s functions are

$$\begin{aligned} & s_{\rm PC} := \zeta^4(1 - 2u + v^2) - 4M^2\zeta^2 , \\ & s_{\rm D} := (\zeta^2 + M^2)(\zeta^2 + m^2 - 2u\zeta^2) + v^2\zeta^4 , \\ & s_{\rm BS} := \zeta^4(1 - 2u + v^2) - 2M^2\zeta^2(u - v) , \\ & s_{\approx \rm BS} := s_{\rm BS} - 2\mu^2[(1+v)\zeta^2 + 2M^2] . \end{aligned} \tag{3.4}$$

For later use, let us note that Eqs. (3.1) and (3.2) imply the relations

$$s \ge 0, \quad |v| \le u . \tag{3.5}$$

IV. PAIR CREATION

A. Total transition probabilities

For the PC, BS, and $\approx$BS processes the triangle inequalities (2.8) are not satisfied. Using the appropriate formulas of the previous sections, we find that for all these processes the total transition probability $w(j)$ can be written in terms of the u, v variables in the form

$$w(j) = \frac{\lambda^2 \sin^2(\pi/b)}{16\pi^2 b} LTI_l(s) \tag{4.1}$$

where, for each process, the s function has to be chosen appropriately from the expressions listed in Eq. (3.4), while

$$I_l(s) := \int_0^1 du \int_{-u}^{u} dv \frac{\coth(w/b)[\exp(-2|l|w_1/b) + \exp(-2|l|w_2/b)]H(s)}{[\sinh^2(w/b) + \sin^2(\pi/b)](1 - 2u + v^2)\sqrt{s}} \tag{4.2}$$

with

$$\cosh w_1 = \frac{1 - v}{\sqrt{2(u - v)}} , \quad \cosh w_2 = \frac{1 + v}{\sqrt{2(u + v)}} , \quad \cosh w = \frac{1 - u}{\sqrt{u^2 - v^2}} . \tag{4.3}$$

For the pair-creation process an additional multiplicative factor ($\frac{1}{2}$) is to be understood in Eq. (4.1), because the created particles are identical. Note that because of the factor $\sin^2(\pi/b)$, the total probability vanishes when the cosmic-string parameter $b^{-1} = n \in \mathbb{N}$.

In the following we will deal with the pair-creation process while the bremsstrahlung process will be studied in a subsequent paper.

Pair creation is a process that cannot happen in empty Minkowski space. The presence of a string disturbs the Minkowski space in such a way that there is now a well-defined linelike center-breaking Lorentz symmetry, whereas spacetime remains locally flat outside the string. The first fact has the consequence that pair creation as a nonlocal quantum-mechanical process is now allowed. Stressing on the other hand the classical traits, one may imagine this process as a somewhat localized decay of a massless particle into two massive ones. The probability for this localized decay to occur should vary with the distance from the string. Although our scheme is based as usual on transitions between quantum states which cover nearly the whole space, a physical understanding of the space dependence of the decay process can be obtained. For this we will refer to the discussion in paper I and study the total transition probabilities as well as the localized transition probability in Sec. II.

In general the integral of Eq. (4.2) cannot be calculated analytically. To do so we have to restrict to limiting cases. Thereby, instead of describing the state of the decaying particles by the z angular momentum quantum label $l \in \mathbb{Z}$, we use the discrete radius of closest approach

$$r_{\min} = \frac{|l|}{b\zeta} , \tag{4.4}$$

together with the z momentum κ, and the measure of the x-y momentum ζ. The interior of the cylinder $r \lesssim r_{\min}$ is the region of localized absence of the decaying particles. In the following we consider decaying massless particle modes in the limit of low and high energies for small and large values of $r_{\min}$. Although the equations below are valid for arbitrary values κ of the z momentum of the massless particles, we refer in the discussion to the case $\kappa = 0$ so that ζ represents the energy of the decaying particles.

B. Low-energy behavior

In the low-energy limit the energy ζ of the decaying particles has values close to the mass threshold $\zeta = 2M$, fixed by the mass M of the produced massive particles. For this case Harari and Skarzhinsky have been able to work out in Refs. [12] and [13] the integrals in (4.2) analytically. In accordance with their results we obtain the following expression for the total transition probability for $\zeta \approx 2M$:

$$\frac{w_{\mathrm{PC}}(\kappa,l,\zeta)}{LT}=\left[\frac{\lambda}{2M}\right]^2\frac{\sqrt{2\pi}\sin^2(\pi/b)}{4\pi^2 b}\left[1-\frac{2M}{\zeta}\right]^{(|l|+2)/b+3/2}\frac{\Gamma[1+(|l|+1/b)]\Gamma[1+(1/b)]}{\Gamma[5/2+(|l|+2/b)]}\;. \tag{4.5}$$

Introducing the Compton wavelength $\lambda_C=M^{-1}$ of the decay products and using Eq. (4.4), we may replace in this limit the angular momentum quantum label l according to

$$\frac{|l|}{b}\approx\frac{2r_{\min}}{\lambda_C}\;. \tag{4.6}$$

Equation (4.5) then shows that for states with a radius of closest approach $r_{\min}$ which is smaller than the Compton wavelength ($r_{\min} \ll \lambda_C$), the total transition probability w_{PC} approaches a value which is independent of $r_{\min}$:

$$\frac{w_{\mathrm{PC}}(\kappa,r_{\min},\zeta)}{LT}\to\left[\frac{\lambda}{2M}\right]^2\frac{\sqrt{2\pi}\sin^2(\pi/b)}{4\pi^2 b}\left[1-\frac{2M}{\zeta}\right]^{2/b+3/2}\frac{\{\Gamma[1+(1/b)]\}}{\Gamma[5/2+(2/b)]}\;,\quad \zeta\approx 2M\;. \tag{4.7}$$

On the other hand, for states with large $r_{\min}(r_{\min} \gg \lambda_c)$, we obtain from (4.5) that w_{PC} falls off faster than exponentially with increasing $r_{\min}$. We will interpret these results below.

C. High-energy behavior

To begin with, we discuss states with small $r_{\min}$. In this case we can treat the total probabilities for two different high-energy domains. From Eq. (4.1) we find, using Eq. (A10) with (A7) and (A12) of the Appendix, that without any restriction of the value of the string parameter b we have in the *ultrahigh-energy domain*

$$\frac{\zeta}{2M}\gg\frac{1}{b|\sin(\pi/b)|}\;, \tag{4.8}$$

approximately

$$\frac{w_{\mathrm{PC}}(\kappa,r_{\min},\zeta)}{LT}=\frac{1}{24\pi^2}\left[\frac{\lambda}{2M}\right]^2\left[1-3\pi b\frac{r_{\min}}{\lambda_C}+O\left[\frac{r_{\min}^2}{\lambda_C^2}\right]\right]. \tag{4.9}$$

This means that for $r_{\min} \ll \lambda_c$ the transition probability becomes again independent of $r_{\min}$ and furthermore also of b and of the energy ζ.

If one specifies the string parameter b, additional information can be obtained. For the GUT cosmic string with $b \approx 1$ and therefore $1 \ll (1-b)^{-1}$, the right-hand side (RHS) of Eq. (4.8) becomes $\pi^{-1}(1-b)^{-1}$. For such values of b we can in addition to the domain (4.8) treat the *less-high-energy domain*

$$1\ll\frac{\zeta}{2M}\ll\frac{1}{\pi(1-b)}\;. \tag{4.10}$$

Using in (4.1) Eq. (A10) with (A8) and (A13) of the Appendix we now obtain

$$\frac{w_{\mathrm{PC}}(\kappa,r_{\min},\zeta)}{LT}=\frac{(1-b)^2}{840}\left[\frac{\lambda}{2M}\right]^2\left[\frac{\zeta}{M}\right]^2\times\left[1-\frac{7\pi b}{4}\left[\frac{r_{\min}}{\lambda_c}\right]+O\left[\frac{r_{\min}^2}{\lambda_C^2}\right]\right]. \tag{4.11}$$

Again the transition probability turns out to be independent of $r_{\min}$ for $r_{\min} \ll \lambda_C$. But for this energy domain it becomes proportional to $(1-b)^2$ and $(\zeta/M)^2$.

To complete the high-energy discussion, we turn to decaying states with a large radius of closest approach $r_{\min}$. As shown in the Appendix, for high energies, the main contribution to the total probability comes from processes for which the quantities w_1 and w_2 of Eq. (4.3) satisfy

$$w_1\approx w_2\approx\frac{2M}{\zeta}\ll 1\;. \tag{4.12}$$

Returning to (4.2), this implies that w_{PC} shows an exponential fall off with $r_{\min}/\lambda_C$ for increasing $r_{\min} \gg \lambda_C$.

Taking the results together, we find for high energies of the pair-producing massless particles, as well as for low energies near the mass threshold, a similar dependence of the decay probability on the radius of closest approach $r_{\min}$ of the decaying state: For $r_{\min}$ smaller than the Compton wavelength of the created massive particles ($r_{\min} \ll \lambda_c$), the creation probability is insensitive to a variation of $r_{\min}$. For large $r_{\min} \gg \lambda_C$ it falls off exponentially, or even faster, with $r_{\min}$.

D. Localization

To interpret this, we refer to the suggestive picture of the localized decay of classical particles and take into account the result of paper I that there is an effective interaction region around the cosmic string. Because of the phenomenon of localized absence, this lies for a particular quantum state outside a cylinder around the cosmic string with radius $r_{\min}$ which is also essentially the radius of closest approach to the string for the decaying particles. Assuming that in a string spacetime the non-Minkowskian decay of the massless particles is induced most effectively if these particles are as close as possible to the string, then, for given impact parameter a, these decays are most likely to happen for particles near the cylinder of radius $r_{\min}$ (compare the localization discussion in paper I). But it is important that the resulting creation probability depends, in addition, on the Compton wavelength of the created particles which represents an "extension" of these particles. For $\lambda_C \gg r_{\min}$, the produced particles, which before their creation can be considered as virtual pairs, clearly "extend" over the point $r=0$ where the string is localized. As long as this is the case, the decay probability remains insensitive to a change of $r_{\min}$ (meaning a change of the radius of the main decay region).

On the other hand, for $r_{\min} \gg \lambda_C$, the produced particles do not extend up to the string at the moment of their production. In this case the actual place of the production becomes important and the decay probability decreases exponentially with the distance from the string. To support this interpretation, we turn to a discussion of the localization based on the Eq. (2.14).

E. Second approach to localization

The quantity $\delta w_{\mathrm{PC}}(r_l;\zeta,\kappa)$ of Eq. (2.14) can be obtained in the limiting case of the two high-energy regions for which the total probability was obtained as well.

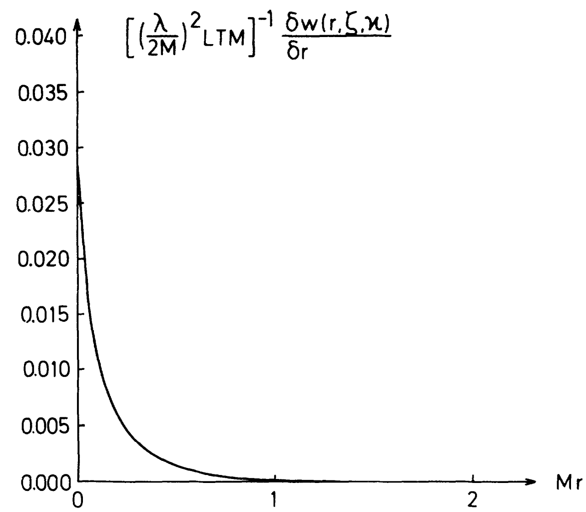

FIG. 2. The local distribution of the pair-creation probability.

For distances r_l from the string which are smaller than the Compton wavelength λ_C of the created particles ($r_l \ll \lambda_C$), we find that Eq. (2.14), using Eqs. (4.1), (4.9), and (4.11), gives

$$\frac{\delta w_{\mathrm{PC}}(r_l;\zeta,\kappa)}{LT\delta r_l} = \begin{cases} \dfrac{1}{8\pi}\left[\dfrac{\lambda}{2M}\right]^2 M & \text{for } \dfrac{b\zeta}{2M} \gg \dfrac{1}{|\sin(\pi/b)|}\,, \\[2ex] \dfrac{(1-b)^2\pi}{480}\left[\dfrac{\lambda}{2M}\right]^2 \dfrac{\zeta^2}{M} & \text{for } 1 \ll \dfrac{\zeta}{2M} \ll \dfrac{1}{\pi(1-b)}\,. \end{cases} \tag{4.13}$$

This is constant for fixed energy ζ and string parameter b. As compared with the constancy in the ultrahigh-energy domain (first equation), there is in the less-high-energy domain (second equation) an increase with ζ^2 and a $(1-b)^2$ dependence. The result (4.13) shows that for fixed energy every thin cylindrical ring with thickness $\delta r_l = (b\zeta)^{-1}$ contributes with the same amount to the creation probability as long as the created particles have an extension λ_C overlapping the string. The localized probability per cylindrical ring is constant.

For regions not so close to the string we obtain a numerical result which is the smoothed out curve of Fig. 2. It refers to a fixed energy ζ of the massless particles with $M/\zeta = 0.005$ and $b=0.8$. The figure shows that for increasing distance r from the string the contribution of a cylindrical ring to the total transition probability rapidly decreases. For $r_l \gg \lambda_C$ there is essentially no contribution, meaning that no pair creation is happening in these regions. These results reflect the interpretation given above.

F. Cross section

We finally turn to an independent approach to localization and discuss pair creation for a differently prepared initial state of massless particles. For an ingoing plane wave with momentum $\mathbf{p}$, the cross section per unit z length gives a measure of the total extension of the effective interaction region. Using Eqs. (2.17), (4.1) and Eqs. (A16), (A17) of the Appendix we find that

$$\sigma_{\mathrm{PC}}(\mathbf{p}) = \begin{cases} \dfrac{1}{192\pi}\left[\dfrac{\lambda}{2M}\right]^2 \dfrac{1}{M} & \text{for } \dfrac{b\zeta}{2M} \gg \dfrac{1}{|\sin(\pi/b)|}\,, \\[2ex] \dfrac{3\pi(1-b)^2}{17920}\left[\dfrac{\lambda}{2M}\right]^2 \left[\dfrac{\zeta}{M}\right]^2 \dfrac{1}{M} & \text{for } 1 \ll \dfrac{\zeta}{2M} \ll \dfrac{1}{\pi(1-b)}\,, \end{cases} \tag{4.14}$$

where $\zeta^2=p_x^2+p_y^2$. In the limit of GUT cosmic strings this result agrees with the one of Harari and Skarzhinsky in Ref. [13] apart from an additional factor $8\pi^4$ which appears in (4.14).

In the ultrahigh-energy limit, the $\sigma_{\rm PC}(\mathbf{p})$ turns out to be proportional to the Compton wavelength. In the less-high-energy limit we find in addition again the typical dependence on $(1-b)^2$ and ζ^2. Note that this conclusion leads to an interaction region which agrees with the one discussed in the previous subsections.

V. DECAY OF A MASSIVE PARTICLE

We turn to a physically different situation and consider the decay process D of Sec. III, an example of a process which is allowed in Minkowski spacetime.

As can be readily seen from Fig. 1, the total probability $w_{\rm D}$ of the decay process of Sec. III contains processes which violate the triangle inequalities (2.8), and others which do not. We will restrict ourselves here to the contribution $w_{\rm D}^{(t)}$ of the latter type of processes, which we will call type (t) process. With Eqs. (2.4)–(2.7), (2.12), and (2.13) we find that in terms of the u,v variables of Eq. (3.2), the $w_{\rm D}^{(t)}$ is

$$w_{\rm D}^{(t)}(j)=\frac{\lambda^2 TL}{8b}\frac{b\gamma R}{4\pi^2}\frac{\zeta}{2\hat{\xi}} \times\int_{-\hat{\xi}/\zeta}^{\hat{\xi}/\zeta}dv\int_{\beta(v)}^{\alpha(v)}du\frac{1}{\sqrt{(u-\beta)(\alpha-u)}}\;, \tag{5.1}$$

where

$$2\alpha(v)=\left[\frac{\hat{\xi}}{\zeta}\right]^2+v^2\left[\frac{\zeta}{\hat{\xi}}\right]^2,\quad 2\beta(v)=1+v^2\;, \tag{5.2}$$

and γ is an unspecified dimensionless parameter. The integral in (5.1) is elementary and we obtain

$$w_{\rm D}^{(t)}(j)=TL\frac{\lambda^2\gamma R}{32\pi\zeta}\;. \tag{5.3}$$

We observe here that the result is independent of b and is proportional to the normalization radius R.

As we have already remarked above, in general, that is, for $b^{-1}\neq n\in\mathbb{N}$, the total probability $w_{\rm D}$ contains also contributions from processes which violate the triangle inequalities. Using numerical calculations we find that these contributions diverge with R, but less rapidly than the $w_{\rm D}^{(t)}$.

Let us now return to Eq. (5.1) from which we can obtain the lifetime $\tau^{(t)}$ of the decaying particles via the "decay channel" of type (t) processes. Remembering that the particle density of our cylindrical states $|1_{\kappa l\zeta}\rangle$ is $E_j/(\pi b r\zeta)$ for $r\gg r_{\min}=|l|/(b\zeta)$, we conclude that the total number of decaying particles within a normalization cylindrical box of length L and radius R is, in the limit L and $R\rightarrow\infty$, approximately equal to

$$\int^{(L,R)}br\,dr\,dz\,d\varphi\frac{E_j}{\pi b r\zeta}\approx\frac{2LRE_j}{\zeta}\;. \tag{5.4}$$

Therefore, the lifetime $\tau^{(t)}$ of each decaying particle satisfies

$$\frac{1}{\tau^{(t)}}:=\frac{w_{\rm D}^{(t)}(j)}{T(2LRE_j\zeta^{-1})}=\frac{\lambda^2\gamma}{64\pi E_j}\;. \tag{5.5}$$

Note that the result again does not depend on the cosmic-string parameter b.

The factor γ that appears in this result can be specified by considering the Minkowski limit $b=1$. In this spacetime, because of the momentum conservation in the x-y plane, only type (t) processes contribute to the total decay probability. Therefore, $w_{\rm D}(j)=w_{\rm D}^{(t)}(j)$ and consequently the (total) lifetime of the decaying particles is $\tau=\tau^{(t)}$. However, in Minkowski spacetime one can easily obtain the lifetime by an independent calculation based on the usual plane-wave states. The value of the lifetime obtained in this calculation agrees with the one above provided we set $\gamma=2$.

A. Localization

Unfortunately, for the type (t) processes we cannot use Eq. (2.14) to study their local behavior. The reason is that the probabilities $w^{(t)}(\kappa,l,\zeta)$ and $w^{(t)}(\kappa,l+1,\zeta)$ are infinite, proportional to R, and agree in the limit of $R\rightarrow\infty$. Therefore it is not clear how their difference, which appears in Eq. (2.14), is to be taken.

However, returning to Eq. (5.3), we note that the final result for the total probability $w_{\rm D}^{(t)}$ is proportional to the range R of the normalization radius. This is due to the fact that the presence of the cosmic string is not necessary for the realization of these processes while, on the other hand, its effect on these processes is expected to decrease as one moves away from the string.

However, although for large radial distances the type (t) process dominate (and therefore $w_{\rm D}\approx w_{\rm D}^{(t)}$), near the cosmic string the processes which violate the triangle inequalities are expected to play an important role. Their radial dependence can be, in principle, roughly derived with calculations based on Eqs. (2.14). Consequently we expect that decay rates, lifetimes and so on, change significantly as we approach the cosmic string.

In closing, let us briefly note that a "good" cross-section definition can not be given in this case. The total probability $w_{\rm D}(\mathbf{p})$, out of which we must construct the cross section according to the usual S-matrix scheme, turns out to be proportional to the volume ($\propto LR^2$) since the (t)-channel processes dominate in an infinite space region. Therefore only an "average" cross section could be given, referring only to those (t) processes that take place far away from the cosmic string. Thus we have in such a definition no information for the D processes near the string where, as we explained above, they are expected to be quite different from those in the $r\rightarrow\infty$ region. Therefore we realize that a quantity which would describe satisfactorily the D process should be a spatially dependent one. This quantity however cannot be a cross section since spacetime dependence does not come out from any usual S-matrix scheme.

ACKNOWLEDGMENTS

This work was supported by the European Community DG XII.

APPENDIX

Here we present details for the calculations of Sec. IV on the high-energy behavior of (a) the total probability $w_{PC}(j)$ and (b) the pair-creation cross section.

In the high-energy regime it is more convenient to work not with the u,v variables of Eq. (3.2) but with the τ and σ ones:

$$\tau := v/u, \quad \sigma := (1-2u+v^2)^{1/2} . \tag{A1}$$

(a) In terms of the τ,σ variables, the integral $I_l(s_{PC})$ of Eq. (4.2) is written as

$$I_l(s_{PC}) = \frac{2}{\zeta^2}\int_0^1 d\tau \int_{2M/\zeta}^1 d\sigma \frac{(1-\sigma^2)\coth(w/b)[\exp(-2|l|w_1/b)+\exp(-2|l|w_2/b)]}{\sigma(\sigma^2-4M^2\zeta^{-2})^{1/2}\Delta(1+\Delta)[\sinh^2(w/b)+\sin^2(\pi/b)]} , \tag{A2}$$

with

$$\Delta := [1-\tau^2(1-\sigma^2)]^{1/2} . \tag{A3}$$

The quantities w, w_1, and w_2 are those in Eq. (3.3) but they are now considered to be functions of τ,σ via the inverse relations of (A1):

$$u = (1-\Delta)/\tau^2, \quad v = (1-\Delta)/\tau . \tag{A4}$$

From (A2) it is obvious that for high energies $\zeta/(2M) \gg 1$ the main contribution to $I_l(s_{PC})$ comes from small values of σ, that is for $\sigma \approx 2M/\zeta \approx 0$. For $\sigma \to 0$, as long as τ is not close to one, we have the following limiting values of the quantities that enter the expression for $I_l(s_{PC})$:

$$\begin{aligned} &\Delta \to \Delta_0 := \sqrt{1-\tau^2}, \quad w \to \frac{\sigma(1+\Delta_0)}{\Delta_0} , \\ &w_1 \to \frac{\sigma\tau}{[2(1-\tau)(1-\Delta_0)]^{1/2}} , \quad w_2 \to \frac{\sigma\tau}{[2(1+\tau)(1-\Delta_0)]^{1/2}} . \end{aligned} \tag{A5}$$

Values of τ close to unity do not contribute in $I_l(s_{PC})$ since they make the exponentials in (A2) tend to zero while the rest of the integrand of $I_l(s_{PC})$ (that is the integrand apart from the exponentials) is integrable at $\tau \to 1$. In fact, the main contribution to $I_l(s_{PC})$ comes from values $\tau \to 0$. In this limit we conclude from (A5) that $w_1 \approx w_2 \approx \sigma \approx 2M/\zeta$.

Now, because of Eq. (A5), the exponents in (A2) are proportional to $-|l|\sigma \approx -2|l|M/\zeta$ and therefore the integral $I_l(s_{PC})$, for $|l| \gg \zeta/(2M)$, falls off exponentially with increasing $|l|$.

On the other hand, for $|l| \ll \zeta/(2M)$ each of the exponentials in (A2) is, to a good approximation, equal to one. Therefore in this case the integral $I_l(s_{PC})$ is approximately equal to the value of $I_{l=0}(s_{PC})$:

$$I_0(s_{PC}) \approx \frac{4b^3}{\zeta^2}\int_0^1 d\tau \int_{2M/\zeta}^1 d\sigma \frac{\Delta_0^2(1+\Delta_0)^{-2}}{\sigma^2(\sigma^2-4M^2\zeta^{-2})^{1/2}[\sigma^2(1+\Delta_0)^2+b^2\Delta_0^2\sin^2(\pi/b)]} . \tag{A6}$$

It is obvious from (A6) that the τ and σ integrations separate whenever one of the two terms in the square brackets of (A6) dominates over the other. In particular if $\sigma \sim (\zeta/2M) \gg [1/b|\sin(\pi/b)|]$, then we can neglect the term proportional to σ^2 in (A6). The integral then simplifies to

$$I_0(s_{PC}) \approx \frac{4b}{\sin^2(\pi/b)\zeta^2}\int_0^1 \frac{d\tau}{(1+\sqrt{1-\tau^2})^2}\int_{2M/\zeta}^1 \frac{d\sigma}{\sigma^2(\sigma^2-4M^2\zeta^{-2})^{1/2}} = \frac{b}{3M^2\sin^2(\pi/b)} . \tag{A7}$$

For GUT cosmic strings where $(1-b) \ll 1$ the Eq. (A7) is valid for the ultrahigh-energy region $(\zeta/2M) \gg 1/\pi(1-b)$. Because for GUT cosmic strings $(1-b)^{-1}$ is several orders of magnitude larger than one, one can furthermore consider the high-energy regime $1 \ll (\zeta/2M) \ll [1/\pi(1-b)]$. In this case the term $\propto \sigma^2$ in the square brackets of Eq. (A6) dominates over the term $\propto \sin^2(\pi/b)$. Thus the $I_l(s_{PC})$ simplifies in this case to

$$I_0(s_{PC}) \approx \frac{4}{\zeta^2}\int_0^1 d\tau \frac{1-\tau^2}{(1+\sqrt{1-\tau^2})^4}\int_{2M/\zeta}^1 \frac{d\sigma}{\sigma^4(\sigma^2-4M^2\zeta^{-2})^{1/2}} = \frac{\zeta^2}{105M^4} . \tag{A8}$$

The first correction to the $I_l(s_{PC})$ for $l \neq 0$ can also be obtained analytically. By expanding the exponentials of (A2) in a Taylor series we see that in the $\sigma \to 0$ limit the quantity in the square brackets of the numerator of Eq. (A2) is

$$2 - \frac{2\sigma|l|(1+\Delta_0)}{b\Delta_0} + O(l^2\sigma^2) . \tag{A9}$$

Therefore

$$I_l(s_{PC}) \approx I_0(s_{PC}) - |l|\delta I(s_{PC}) + O(l^2\sigma^2) , \tag{A10}$$

with

$$\delta I(s_{\rm PC})\approx\frac{4b^2}{\zeta^2}\int_0^1 d\tau\int_{2M/\zeta}^1 d\sigma\frac{\Delta_0(1+\Delta_0)^{-1}}{\sigma(\sigma^2-4M^2\zeta^{-2})^{1/2}[\sigma^2(1+\Delta_0)^2+b^2\Delta_0^2\sin^2(\pi/b)]}\ . \tag{A11}$$

Similar to the calculations of (A7) and (A8) above, we obtain in the ultrahigh-energy limit

$$\delta I(s_{\rm PC})=\frac{\pi}{\sin^2(\pi/b)M\zeta}\ . \tag{A12}$$

In the case where $1\ll(\zeta/2M)\ll(1-b)^{-1}$ we find

$$\delta I(s_{\rm PC})=\frac{\pi\zeta}{60M^3}\ . \tag{A13}$$

(b) In the calculations for the pair-creation cross section we must work out the expression $\sum_{l=-\infty}^{+\infty}I_l(s_{\rm PC})$. Interchanging the summation and integration and performing the summation, we are left with an expression which is given by the $I_l(s_{\rm PC})$ of Eq. (A2) with the quantity between the square brackets of the numerator changed to

$$\frac{\exp(w_1/b)}{\sinh(w_1/b)}+\frac{\exp(w_2/b)}{\sinh(w_2/b)}-2\ . \tag{A14}$$

In the high-energy limit where $\sigma\to 0$, the expression in (A14) goes to $2b/\sigma$. Therefore for $\zeta\gg 2M$ we obtain

$$\sum_{l=-\infty}^{+\infty}I_l(s_{\rm PC})=\frac{4b^4}{\zeta^2}\int_0^1 d\tau\int_{2M/\zeta}^1 d\sigma\frac{\Delta_0^2(1+\Delta_0)^{-2}}{\sigma^3(\sigma^2-4M^2\zeta^{-2})^{1/2}[\sigma^2(1+\Delta_0)^2+b^2\Delta_0^2\sin^2(\pi/b)]}\ . \tag{A15}$$

In the ultrahigh-energy limit we have

$$\sum_{l=-\infty}^{+\infty}I_l(s_{\rm PC})=\frac{\pi b^2\zeta}{24\sin^2(\pi/b)M^3}\ . \tag{A16}$$

For $1\ll(\zeta/2M)\ll(1-b)^{-1}$ we find

$$\sum_{l=-\infty}^{+\infty}I_l(s_{\rm PC})=\frac{3\pi\zeta^3}{2240M^5}\ . \tag{A17}$$

In deriving Eqs. (A7), (A8), (A12), (A13), (A16), and (A17) we have used for the τ integrals the change of variable $\tau=\sin x$ and the relation (3.624.4) of Ref. [16],

$$\int_0^{\pi/4}\frac{\cos^\mu(2x)}{\cos^{2(\mu+1)}x}dx=2^{2\mu}B(\mu+1,\mu+1),\quad {\rm Re}(\mu)>-1$$

while for the σ integrals the change of variable $y=\sqrt{\zeta^2\sigma^2/4M^2-1}$ with $y\in(0,\sqrt{(\zeta^2/4m^2)-1}\to\infty)$ and the relations (3.252.3) and (3.249.1) of Ref. [16]:

$$\int_0^\infty\frac{dy}{(y^2+c)^{n+3/2}}=\frac{(-2)^n}{(2n+1)!!}\frac{\partial^n}{\partial c^n}\frac{1}{c}\ ,$$

$$\int_0^\infty\frac{dy}{(y^2+c^2)^n}=\frac{(2n-3)!!\pi}{2(2n-2)!!c^{2n-1}}\ .$$

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